## 1. Details of Module and its structure

| Module Detail | |
|---|---|
| Subject Name | Physics |
| Course Name | Physics 01 (Physics Part-1, Class XI) |
| Module Name/Title | Unit 5 ,Module 6, Mechanical Equilibrium<br>Chapter 7, System of particles and Rotational Motion |
| Module Id | Keph_10706_eContent |
| Pre-requisites | Kinematics, laws of motion, basic vector algebra |
| Objectives | After going through this module, the learners will be able to:<br>• **Understand** the conditions of equilibrium of a rigid body<br>• **Distinguish** between dynamic and static equilibrium<br>• **Apply** concepts about equilibrium in daily life situations |
| Keywords | Torque, moment of force, couple, right hand palm rule, net force, net torque |

## 2. Development Team

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## 1. UNIT SYLLABUS

**Unit V: Motion of System of Particles and Rigid body**

**Chapter 7: System of particles and Rotational Motion**

Centre of mass of a two-particle system; momentum conservation and centre of mass motion. Centre of mass of a rigid body; Centre of mass of a uniform rod.

Moment of a force; torque; angular momentum; law of conservation of angular momentum and its applications.

Equilibrium of rigid bodies; rigid body rotation and equations of rotational motion; comparison of linear and rotational motions.

Moment of inertia; radius of gyration; values of moments of inertia for simple geometrical objects (no derivation). Statement of parallel and perpendicular axes theorems and their applications.

## 2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS 8 Modules

**The above unit is divided into eight modules for better understanding.**

| | |
|---|---|
| Module 1 | • Rigid body<br>• Centre of mass<br>• Distribution of mass<br>• Types of motion: Translatory, circulatory and rotatory |
| Module 2 | • Centre of mass<br>• Application of centre of mass to describe motion<br>• Motion of centre of mass |
| Module 3 | • Analogy of circular motion of a point particle about a point and different points on a rigid body about an axis<br>• Relation v = rω<br>• Kinematics of rotational motion |
| Module 4 | • Moment of inertia<br>• Difference between mass and moment of inertia<br>• Derivation of value of moment of inertia for a lamina about a vertical axis perpendicular to the plane of the lamina<br>• unit<br>• Radius of gyration<br>• Perpendicular and parallel axis theorems |
| Module 5 | • Torque<br>• Types of torque<br>• Dynamics of rotator motion<br>• $m = I\alpha$ |
| Module 6 | • Equilibrium of rigid bodies<br>• Condition of net force and net torque<br>• Applications |
| Module 7 | • Law of conservation of angular momentum and its applications.<br>• Applications |

| Module 8 | • Rolling on plane surface<br>• Rolling on Horizontal<br>• Rolling on inclined surface<br>• Applications |
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## Module 6

### 3. WORDS YOU MUST KNOW

Let us remember the words we have been using in our study of this physics course.

- Rigid body: An object for which individual particles continue to be at the same separation over a period of time
- Point object: A point object is much smaller than the distance due to change in its position.
- Distance travelled: The distance an object has moved from its starting position. Its SI unit is m and it can be zero or positive.
- Displacement: The distance an object has moved from its starting position moves in a particular direction. Its SI unit is m and it can be zero, positive or negative.
- Speed: Rate of change of position and its unit m/s.
- Average speed: Average speed = $\frac{\text{Total path length}}{\text{Total time interval}}$

  Its unit is m/s.
- Velocity (v): Rate of change of position in a particular direction and its unit is m/s.
- Instantaneous velocity The velocity at an instant is defined as the limit of the average velocity as the time interval $\Delta t$ becomes infinitesimally small velocity at any instant of time
- Uniform motion: object covers equal distance in equal interval of time
- Non uniform motion: object covers unequal distance in equal interval of time
- Acceleration (a): rate of change of velocity with time and its unit is m/s$^2$. Velocity may change due to change in its magnitude or change in its direction or change in both magnitude and direction.
- Constant acceleration: Acceleration which remains constant.

- Momentum (p): The impact capacity of a moving body is mv and its unit is kg m/s.
- Force (F): Something that changes the state of rest or uniform motion of a body. Unit of force is Newton. It is a vector, as it has magnitude which tells us the strength or magnitude of the force and the direction of force is very important
- Constant force: A force for which both magnitude and direction remain the same with passage of time
- Variable force: A force for which either magnitude or direction or both change with passage of time
- External unbalanced force: A single force or a resultant of many forces that act externally on an object.
- Dimensional formula: An expression which shows how and in which way the fundamental quantities like, mass (M), length (L) and time (T).
- Kinematics: Study of motion without involving the cause of motion
- Dynamics: Study of motion along with the cause producing the motion
- Vector: A physical quantity that has both magnitude and direction .displacement is a vector, force is a vector, acceleration is a vector etc.
- Vector algebra: Mathematical rules of adding, subtracting and multiplying vectors
- Resolution of vectors: A vector can be resolved in two mutually perpendicular directions. We used this for vector addition and in our study of motion in 2 and 3 dimensions.
- Dot product: two vectors on multiplication yield a scalar quantity. Dot product of vector A and B: A.B= $|A||B|cos\theta$ where θ is the angle between the two vectors. Dot product is a scalar quantity and has no direction. It can also be taken as the product of magnitude of A and the component of B along A or product of B and component of A along B.
- Work: Work is said to be done by an external force acting on a body if it produces displacement W= F.S cosθ, where work is the dot product of vector F( force) and Vector S (displacement) and θ is the angle between them . Its unit is joule and dimensional formula is $ML^2T^{-2}$. It can also be stated as the product of component of the force in the direction of displacement and the magnitude of

displacement. Work can be done by constant or variable force and work can be zero, positive or negative.

- Energy: The ability of a body to do work. Heat, light, chemical, nuclear, mechanical are different types of energy. Energy can never be created or destroyed it only changes from one form to the other.
- Kinetic Energy: The energy possessed by a body due to its motion = ½ $mv^2$, where 'm' is the mass of the body and 'v' is the velocity of the body at the instant its kinetic energy is being calculated.
- Conservative force: A force is said to be conservative if the work done by the force in displacing a body from one point to another is independent of the path followed by the particle and depends on the end points. Example: gravitational force.
- Non- conservative forces: **A** force is said to be non-conservative if: the work done by it on an object depends on the path and the work done by it through any round trip is not zero. Example: friction.
- Work Energy theorem: Relates work done on a body to the change in mechanical energy of a body i.e., $W = F.S = ½ mV_f^2 - ½ mV_i^2$
- Conservation of mechanical energy: Mechanical energy is conserved if work done is by conservative forces.
- Potential energy due to position: Work done in raising the object of mass m to a particular height (distance less than radius of the earth) = 'mgh'.
- Collision: Sudden interaction between two or more objects. We are only considering two body collisions.
- Collision in one dimension: Interacting bodies move along the same straight path before and after collision.
- Elastic collision: Collision in which both momentum and kinetic energy is conserved.
- Inelastic collision: Momentum is conserved but kinetic energy is not conserved.
- Coefficient of restitution: The ratio of relative velocity after the collision and relative velocity before collision. Its value ranges from 0-1.
- Torque: It is rotational analogous of Force and it has following characteristics:

i. Torque is the turning effect of the force about the given axis of rotation.
ii. The torque equals the moment of the force about the given axis/ point of rotation.
iii. Just as force equals the rate of change of linear momentum, torque equals the rate of change of angular momentum.

## 4. INTRODUCTION TO THE CONCEPT OF MECHANICAL EQUILIBRIUM

"Balance" is what we all are striving for in our daily life; we are trying to harmonize the needs and requirements of family and friends and society in our personal and social life.

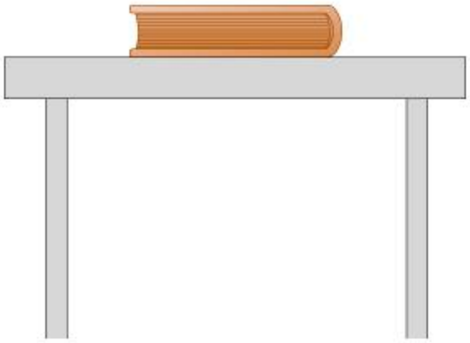
On the monetary front we try to balance our expenses with the earnings.

These examples indicate our pursuit for the balance, or equilibrium in different facets of our life.

In the physical world around us there are various kinds of phenomenon which, take place to attain a state of equilibrium. This is as if a state is to be attained, where the existing quantities involved can neutralize each other. A thermometer acquires the same temperature as that of the object being measured, when it has attained a state of thermal equilibrium. You might have heard of chemical equilibrium in chemistry where the rate of the forward reaction is equal to the rate of the reverse reaction. Different systems seem to have a tendency to attain equilibrium of different kinds. One can say out that in equilibrium, there is a sort of "balance", in the given state of that system; whether it may be money, in case of financial equilibrium, heat in case of thermal equilibrium, rate of reactions, in case of chemical equilibrium, and so on.

**In this module we will be concerned with mechanical equilibrium**. A book on a table top, picture on the wall fan from a ceiling ,small rocks placed one over other, yet precariously balanced etc are common examples of mechanical equilibrium.

If one observes, they are all in a state of rest. This suggests that the different forces, acting on the constituents must be such that the net force zero. However, we have also learnt that there are systems, that are in motion; yet the net force, on them, is zero.



In general, we can then say, that a system is said to be in mechanical equilibrium when its state of motion doesn't change i.e. the system continues to remain at rest, or in its state of uniform motion. One can say that the term mechanical equilibrium implies either the object is at rest and stays at rest is said to be in static mechanical equilibrium or the center of mass, of the system, moves with a constant velocity is said to be in dynamic mechanical equilibrium.

Static equilibrium represents a common situation in engineering practice, and the principles it involves are of special interest to civil engineers, architects, and mechanical engineers. The bridges, buildings etc are to be kept intact, and static, under the influences of many forces acting on them. The principles of equilibrium are very often used in designing them and explain their conditions.

## 5. CONDITIONS OF MECHANICAL EQUILIBRIUM

We now know that when a particle is in mechanical equilibrium—the particle does not accelerate—in an inertial frame of reference, when the vector sum of all the forces, acting on the particle is zero. For an extended body, the equivalent statement is that the center of mass of the body has zero acceleration; this happens when if the vector sum of all external forces acting on the body is zero. This is often called the **first condition for equilibrium**. It is also called the condition for the **translatory equilibrium**. In vector and component forms, we can write:

$$\overrightarrow{\mathbf{F}_{\mathbf{net}}} = \mathbf{0} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 1$$

$\mathbf{F}_{\mathbf{x,net}} = \mathbf{0}$, $\quad \mathbf{F}_{\mathbf{y,net}} = \mathbf{0} \quad$ **and** $\quad \mathbf{F}_{\mathbf{z,net}} = \mathbf{0}$

A **second condition for an extended body to be in equilibrium** is that the body must have no tendency to rotate. This condition is based on the dynamics of rotational motion in exactly the same way as the **first condition is based on Newton's first law**. A rigid body that, in an inertial frame, is not rotating about a certain point has zero angular momentum about that point. We can also think of a rotating body whose angular momentum does not change with time, i.e., whose angular momentum stays constant. In either of such cases, the sum of torques due to all the external forces acting on the body must be zero. This is the second condition for equilibrium. It is also called the condition for the **rotational equilibrium**. In vector and component forms

$$\overrightarrow{\boldsymbol{\tau}_{\mathbf{net}}} = \mathbf{0}\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad 2$$

$$\boldsymbol{\tau}_{\mathbf{x,net}} = \mathbf{0}, \qquad \boldsymbol{\tau}_{\mathbf{y,net}} = \mathbf{0} \qquad \textbf{and} \qquad \boldsymbol{\tau}_{\mathbf{z,net}} = \mathbf{0}\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

One may raise a question, whether the rotational equilibrium condition remains valid, if the origin, with respect to which the torques is taken, is shifted.

One can show that if the rotational equilibrium condition $[\overrightarrow{\boldsymbol{\tau}_{\mathbf{net}}} = \mathbf{0}]$ holds for a rigid body, then such a shift of origin does not matter, i.e. the rotational equilibrium condition is independent of the location of the origin, or the point about which the torques are taken.

The above equations 1 and 2 give six independent conditions to be satisfied for mechanical equilibrium of a rigid body,

In a real life situation, in case, all the forces, acting on the body, are coplanar, then we need only three conditions to be satisfied for mechanical equilibrium.

**Two of these conditions** correspond to translational equilibrium; the sum of the components of the forces, along any two perpendicular axes in the plane must be zero. If the object lies in the X-Y plane, the translatory equilibrium conditions are:

$$\mathbf{F}_{\mathbf{x,net}} = \mathbf{0}, \qquad \mathbf{F}_{\mathbf{y,net}} = \mathbf{0}$$

The **third condition** corresponds to rotational equilibrium. The sum, of the components of the torques along any axis perpendicular to the plane of the forces must be zero. If XY is this plane, the torque, about the z axis must be zero.

$$\boldsymbol{\tau}_{\mathbf{z,net}} = \mathbf{0}$$

These conditions, of **equilibrium of a rigid body**, are more exhaustive as compared with that for a particle, considered in earlier chapters. Since rotational motion is not considered, for a particle the conditions of rotational equilibrium are not required for a particle. Therefore, only the conditions for translational equilibrium, ($\overrightarrow{F_{net}} = 0$) apply to a particle.

**Thus, for equilibrium of a particle, the vector sum of all the forces on it must be zero. Since all these forces act on the single particle, they must be concurrent.**

A body, or a system, may also be in partial equilibrium, i.e., it may be in translational equilibrium and not in rotational equilibrium, or it may be in rotational equilibrium and not in translational equilibrium.

**THINK ABOUT THESE**

1. **Can a body be in equilibrium if only one external force acts on it? Explain.**
2. **Can a body be in equilibrium even if it is not at rest? Explain.**
3. **If you find the net torque, and the net force, on a system to be zero, (a) could the system still be rotating, with respect to you? (b) Could it still be translating, with respect to you?**

## 6. CENTRE OF GRAVITY

For every object, on the earth's surface, the gravitational force, due to the earth, is acting every moment this force is trying, not only, to accelerate the objects them towards it but in case of extended objects, it may try to rotate them also. **Center of gravity is that point in a given body, around which the resultant torque, due to these gravity forces, vanishes.**

The concept can be useful in designing structures, especially buildings, bridges etc., so that they remain stable under the influence of the forces acting on them. It can be also helpful in understanding the behavior of a moving body, when it is acted on by "gravity forces".

This idea can be easily demonstrated using a ruler. Put the ruler on one finger and move it along until it 'balances'. Your finger is then under the 'centre of gravity' of the ruler. This can be done with other small objects also to find their 'centre of gravity'.

The centre of gravity is not necessarily the geometric centre of the object: try it with a spoon! If you balance it in the centre, the weight of the spoon end, will pull that end down and you will probably drop it. The centre of mass is closer to the heavy end, so that the weights, on both its sides, are equal.

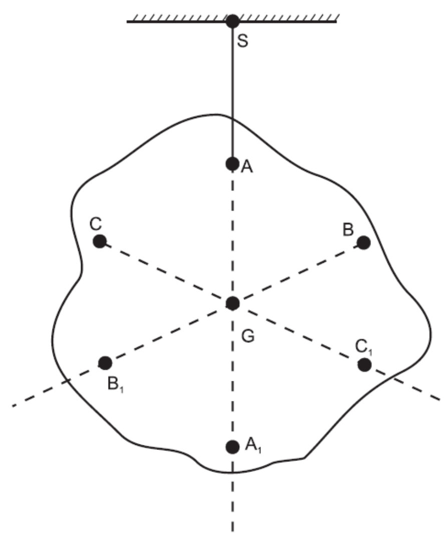
The object balances at its centre of gravity because the weight on each side is equal. To be more precise, the moments caused by the weight on each side, balance out;

the object is then in equilibrium, and hence does not move.

One can easily determine the location of centre of gravity of an arbitrary shaped planar object. This can be done as follows:

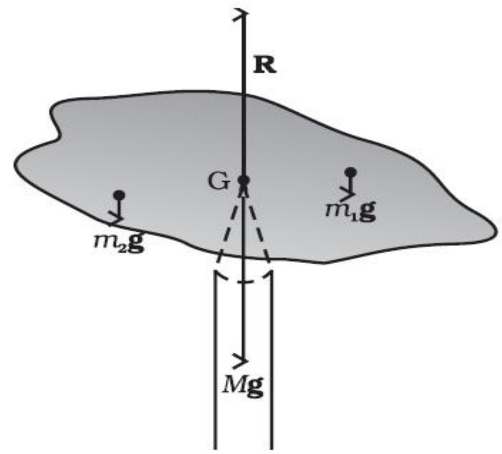
If we just balance the planar object, using a string, or an edge, the point at which the object is balanced is the center of gravity. (Just like balancing a pencil on your finger!)

Another way is a two step method shown below. In Step 1, hang the planar object from any point, say A and draw a line, on the object along the string; let us say it is $AA_1$. For Step 2, repeat the procedure from another point, like B or C, on the object. The intersection, of the lines gives the location of the center of gravity of the given object; here it is shown as G. This procedure works well for irregularly shaped objects that are, otherwise, hard to balance.




How can we find the location of the centre of gravity (CG)? Take an irregular shaped cardboard and a narrow tipped object like a pencil. You can locate, by trial and error, a point G on the cardboard where it can be balanced on the tip of the pencil. (The cardboard remains horizontal in this position.) This point of balance is the centre of gravity (CG) of the cardboard. For a non planar, three dimensional object, one requires three points of suspension, not all lying in the same plane.

The tip of the pencil provides a vertically upward force R due to which the cardboard is in mechanical equilibrium. As shown in the figure, the reaction of the tip is equal and opposite to Mg, the total weight of (i.e., the force of gravity on) the cardboard and hence the cardboard is in translational equilibrium.




$$\mathbf{R = Mg}$$

It is also in rotational equilibrium; if it was not so, due to the unbalanced torque it would tilt, rotate and eventually fall.

There is torques on the card board due to the forces of gravity, like $m_1g$, $m_2g$ …. etc., acting on the individual particles that make up the cardboard; these torques add up to zero when the card board is in equilibrium.

The CG, of the cardboard, is so located that the total torque on it due to the forces $m_1$ g, $m_2g$ …. etc. is zero. If $\vec{r_i}$ is the position vector of the i th particle of an extended body with respect to its CG, the torque about the CG, due to the force of gravity on this particle, is given by:

$$\vec{\boldsymbol{\tau}_i} = \vec{\mathbf{r}_i} \times \mathbf{m_i}\vec{\mathbf{g}}$$

The total gravitational torque about the CG is zero, i.e.

$$\sum_{i=1}^{N} \vec{\boldsymbol{\tau}_i} = \sum_{i=1}^{N} \vec{\mathbf{r}_i} \times \mathbf{m_i}\vec{\mathbf{g}} = \mathbf{0} \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

We may therefore, define the CG of a body as that point where the total gravitational torque

on the body is zero. We notice that, in the above equation, $\vec{g}$ , acceleration due to gravity, is the same for all particles, and hence it comes out of the summation $\sum$.

$$\therefore \left( \sum_{i=1}^{N} \vec{r_i} \times m_i \right) \vec{g} = 0$$

$$\sum_{i=1}^{N} \vec{\mathbf{r}_i} \times \mathbf{m_i} = \mathbf{0} \quad \ldots\ldots\ldots\ldots\ldots \quad (4)$$

The position vectors $\vec{r_i}$, here are taken with respect to the CG that is CG is the origin.

The position vector, of the center of mass, of a given system, is given by:

$$\overrightarrow{r_{CM}} = \frac{\sum_{i=1}^{N} \vec{r_i} \times m_i}{\sum_{i=1}^{N} m_i} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

From equation (4) and (5), we then get:

$$\overrightarrow{\mathbf{r_{CM}}} = \mathbf{0}$$

The origin must be the centre of mass of the body. Thus, the centre of gravity of the body coincides with the centre of mass in a uniform gravity, or in a gravity-free space. The centre of mass, of a rigid body, is fixed in relation to its constituent particles. However, the centre of gravity will shift if the value of $\vec{g}$ varies from point to point. When the body is small, acceleration due to gravity $\vec{\mathbf{g}}$ does not vary from one point of the body to the other. If the body is so extended that acceleration due to gravity varies from part to part of the body, we can't take out, $\vec{\mathbf{g}}$ as common in equation (3). As a result, the centre of gravity and centre of mass will not coincide. The two, therefore, do not always coincide, however. For example, the Moon's centre of mass is very close to its geometric centre it is not exact because the Moon is not a perfect uniform sphere), but its centre of gravity is slightly displaced toward Earth because of the stronger gravitational force on the Moon's near side.

Basically, the two are different concepts.

The centre of mass has nothing to do with gravity. It depends only on the distribution of mass of the body. The two are different as per their definitions. The former, the centre of mass is the point where if an external force acts on the given system, it shows purely a translational motion. The latter, centre of gravity for a given system is the point about which the net torque due to gravitational force acting on its different constituents, is zero; i.e., the system is in rotational equilibrium due to gravity. It is also considered to be the point where the whole weight of the system is effectively acting.

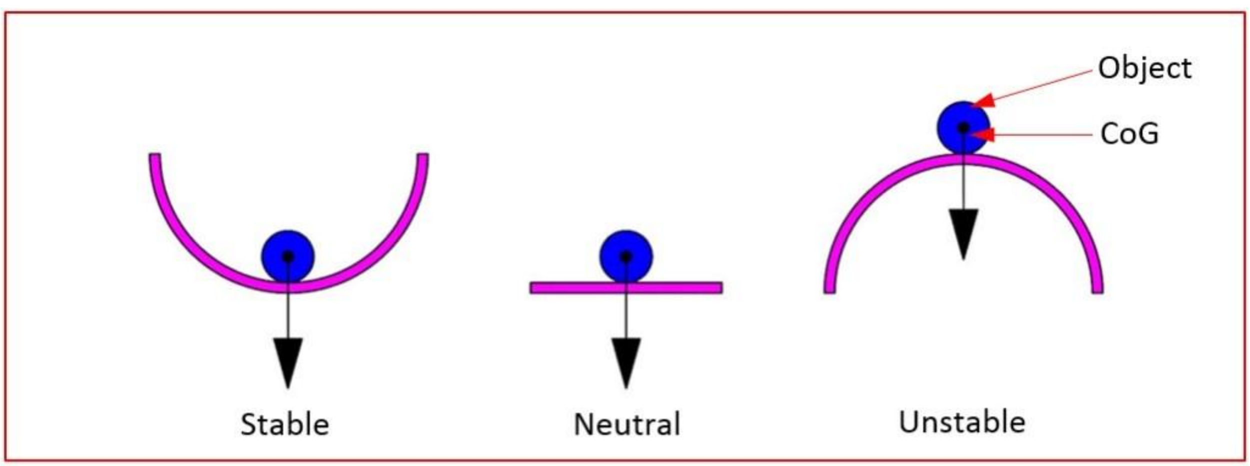
Hence its determination is useful for civil engineers for designing the buildings and bridges which can withstand the different loads and the deforming forces, acting on them. It is also used by aeronautical engineers to design big aircrafts, which can withstand forces during landing and during flight.

The knowledge of CG, of the given system helps them to completely describe the motion of that system through space in terms of the translation of the center of gravity of the object from one place to another and the rotation of the object about its center of gravity if it is free to rotate. If the object is confined to rotate about some other point, like a hinge, we can still describe its motion. In flight aircrafts rotate about their centers of gravity.

## 7. STABLE, UNSTABLE AND NEUTRAL EQUILIBRIUM OF RIGID BODIES

Certain states of balance of rigid bodies are observed to be more resilient, if disturbed the body tries to regain the original state and eventually attains it.

For example a bar pendulum suspended from a point, when disturbed from its equilibrium position, comes back to its original position. However, if the same bar pendulum, standing directly upward from its point of support is slightly disturbed or displaced it departs farther from its original position. The tendency to regain its original equilibrium position can understood with the help of marble placed in a bowl. Consider two situations, first the marble is put inside bowl; and if disturbed it will come back quickly to its original equilibrium position. The marble, when placed above an inverted bowl, if disturbed, will not be able to regain its original position of equilibrium.



If observed carefully the position of the centre of gravity CG plays an important role in the stability of the equilibrium of given body. The lower the centre of gravity (G) is, the more stable the object. The higher it is the more likely the object is to topple over if it is disturbed or pushed. One can argue the above situations in terms of the potential energy of the system. The positions of equilibrium corresponding, to least potential energy are the stable equilibrium positions; the one corresponding to highest potential energy are the unstable equilibrium positions. When a displacement of the body doesn't results in change in its potential energy the equilibrium is said to be neutral.

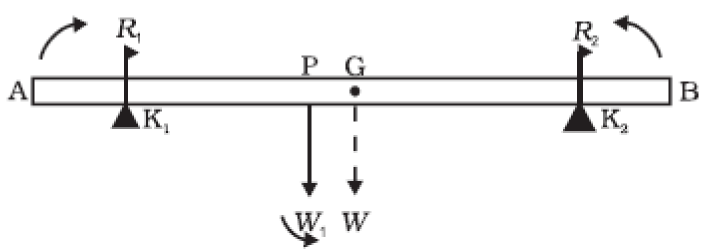
**THINK ABOUT THIS**

**A system is in stable equilibrium. What can we say about its potential energy?**

**EXAMPLE:**

**A metal bar 70 cm long and 4.00 kg in mass, is supported on two knife-edges, placed 10 cm from its each end. A 6.00 kg load is suspended at 30 cm from one end. Find the reactions at the knife edges. (Assume the bar to be of uniform cross section and homogeneous.)**

**SOLUTION:**




The above figure shows the rod AB, the positions of the knife edges $K_1$ and $K_2$ , the centre of gravity of the rod (at G) and the suspended load at P. Note the weight of the rod W acts at its centre of gravity G. The rod is uniform in cross section and homogeneous; hence G is at the centre of the rod; AB = 70 cm. AG = 35 cm, AP = 30 cm, PG = 5 cm, $AK_1$= $BK_2$ = 10 cm and $K_1G$ = $K_2G$ = 25 cm. Also, W= weight of the rod = 4.00 kg and $W_1$= suspended load = 6.00 kg; $R_1$ and $R_2$ are the normal reactions of the support at the knife edges.

**For translational equilibrium** of the rod, the net force, along the vertical direction, must be zero, hence:

$$R_1+R_2-W_1-W=0 \quad \ldots\ldots\ldots\ldots\ldots \qquad (i)$$

(Note that $W_1$ and W act vertically down and $R_1$ and $R_2$ act vertically up.)

For considering rotational equilibrium, we take moments of the forces. A convenient point to take moments about is G. The moments of $R_2$ and $W_1$ are anticlockwise (+ve), whereas the moment of $R_1$ is clockwise (-ve).

**For rotational equilibrium**, of the net torque about any point, (here taken about G), is zero. Hence:

$$-R_1\,(K_1G) + W_1\,(PG) + R_2\,(K_2G) = 0 \quad \ldots\ldots\ldots\ldots \text{(ii)}$$

It is given that W = 4.00g N and $W_1$ = 6.00 g N, where g = acceleration due to gravity.

If we take g = 9.8 m/s$^2$, then on putting numerical values in equation (i) we get

$$R_1 + R_2 - 4.00g - 6.00g = 0$$

or $$R_1 + R_2 = 10.00g = 98.00\text{ N} \quad \ldots\ldots\ldots\ldots \qquad \text{(iii)}$$

From (ii),

$$-\,0.25\,R_1 + 0.05\,W_1 + 0.25\,R_2 = 0$$

$$R_1 - R_2 = (1.2g)\text{ N} = 11.76\text{ N} \ldots\ldots\ldots\ldots\ldots \text{(iv)}$$

From (iii) and (iv)

$$R_1 = 54.88\text{ N} \approx 55\,N$$

$$R_2 = 43.12\text{ N} \approx 43\,N$$

Thus, the reactions, of the supports are nearly 55 N at $K_1$ and 43 N at $K_2$.

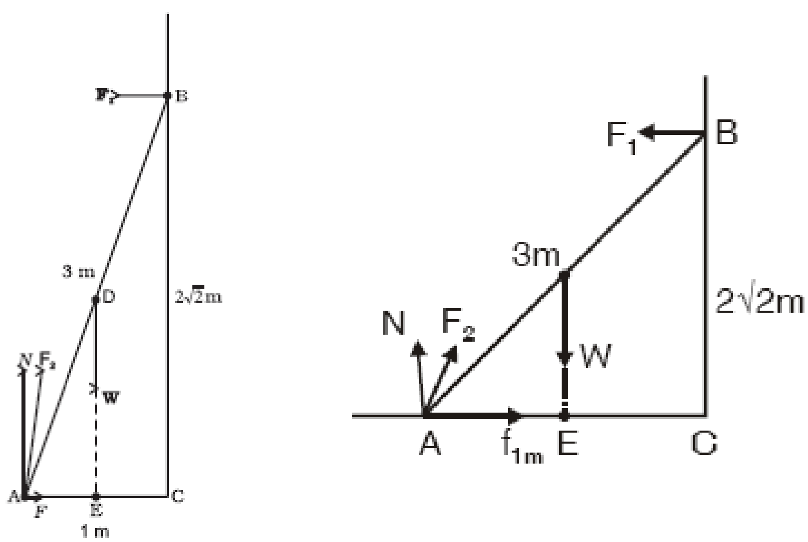
**EXAMPLE:**

**A 3m long ladder weighing 20 kg, leans on a frictionless wall. Its feet rest on the floor, 1 m from the wall, as shown in figure given below. Find the reaction forces of the wall and the floor.**

**SOLUTION:**

The ladder AB is 3 m long, its foot A is at distance AC = 1 m from the wall. From Pythagoras theorem, BC = 2 $\sqrt{2}$ m. The forces on the ladder are its weight W, (acting at its centre of gravity D), and the reaction forces, $F_1$ and $F_2$, of the wall and the floor respectively. Force $F_1$ is perpendicular to the wall, since the wall is frictionless. Force $F_2$ is resolved into two components, the normal reaction N and the force of friction F.

(Note that: the force of friction (F) has to prevent the ladder from sliding away from the wall; it is therefore, directed toward the wall.)




**For translational equilibrium**, we must have $\overrightarrow{\mathbf{F_{net}}} = \mathbf{0}$

Taking the components of the different forces, along the vertical direction, we get

$$N - W = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(i)}$$

Taking the components of forces, in the horizontal direction, we get

$$F - F_1 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(ii)}$$

**For rotational equilibrium**, we must have $\overrightarrow{\boldsymbol{\tau}_{net}} = \mathbf{0}$

Now taking the moments of the forces, or torque, about A, we get

$$2\sqrt{\mathbf{2}}\, F_1 - (1/2)\, W = 0 \ldots\ldots\ldots\ldots\ldots \quad \text{(iii)}$$

Now W = 20 g = 20 × 9.8 N = 196.0 N

From (i) N = 196.0

From (iii) $F_1 = W/(4\sqrt{\mathbf{2}}) = 196.0/4\sqrt{\mathbf{2}} = 34.6$ N

From (ii) $F = F_1 = 34.6$ N

Also, $F_2 = \sqrt{\boldsymbol{F}^2 + \boldsymbol{N}^2} = 199.0$ N

The force $F_{2,}$ makes an angle $\alpha$ with the horizontal

$$\tan\alpha = N/F = 4\sqrt{\mathbf{2}}$$

$$\alpha = \tan^{-1}(4\sqrt{\mathbf{2}}) \approx 80^{O}$$

**TRY YOURSELF:**

**A ladder is resting inclined against a wall. Would you feel safer climbing up the ladder if you were told that:**

**(1) the ground is frictionless but the wall is rough or**

**(2) that the wall is frictionless but the ground is rough? Justify your answer.**

## 8. SUMMARY

**In this module, we have learnt:**

- The term mechanical equilibrium implies either the object is at rest and stays at rest is said to be in static mechanical equilibrium or the center of mass, of the system, moves with a constant velocity is said to be in dynamic mechanical equilibrium.

- First condition for equilibrium: is that the center of mass of the body has zero acceleration; this happens when if the vector sum of all external forces acting on the body is zero. It is also called the condition for the translatory equilibrium. In vector and component forms, we can write:

$$\overrightarrow{F_{net}} = 0$$

- Second condition for an extended body to be in equilibrium is that the body must have no tendency to rotate. It is also called the condition for the rotational equilibrium. In vector and component forms, we can write:

$$\overrightarrow{\tau_{net}} = 0$$

- Center of gravity is that point in a given body, around which the resultant torque, due to the gravity forces, vanishes. The concept can be useful in designing structures, especially buildings, bridges etc., so that they remain stable under the influence of the forces acting on them.